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ON THE BOOTSTRAP MULTIVARIATE EXPONENTIALLY WEIGHTED MOVING AVERAGE (BMEWMA) IN SETTING CONTROL LIMITS AND P-VALUES FOR INTERPRETING OUT OF CONTROL SIGNALS

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ABSTRACT

The effects of control limits that are too narrow increase the rate of false alarms, while those that are too wide may not be able to identify special causes of variability in any given process. It is of this view that control chart methodology that can detect small to moderate shifts in the mean vector should be developed so that the probability of detecting or not detecting false alarm rate should be minimized. The bootstrap multivariate exponentially weighted moving average is proposed in setting control limits, while p-value method was introduced to identify out of control signals.

Keywords: Quality characteristics, BMEWMA control chart, out-of-control signals, control limits, p-values.

INTRODUCTION

Many control charts have been proposed for multivariate data, with the most popular being multivariate Shewhart (Hotelling's X^2 and T^2) chart, the multivariate exponentially- weighted moving average (MEWMA) chart, and the multivariate cumulative sum (MCUSUM) chart (Aparisi *et al*., 2004; Montgomery, 2009; Mahmoud and Maravelakis, 2010). Multivariate Hotelling's T^2 charts monitor T^2 statistics that assume the distance between an observation and the scaled-mean established from the in-control data. It has the capability to identify large shifts only but not very good in identifying small as well as moderate shifts in process mean vector, hence the multivariate cumulative sum (MCUSUM). The multivariate cumulative sum (MCUSUM) method graphs the increasing arithmetic of deviations of the observation principles from a given point against time. A vital quality of the CUSUM method is that, it brings together all the information in the series of observation points. This makes the CUSUM method more responsive even to smaller mean out of control signals (Smiley and Keoagile, 2005; Champ and Jones-Farmer, 2007). However, the problems involve with the use of MCUSUM method is the violation of assumption of multivariate normality, hence the multivariate exponentially-weighted moving average (MEWMA) methods.

The multivariate exponentially weighted moving average (MEWMA) was introduced by (Lowry *et al*., 1992) as an annex of univariate exponentially weighted moving average (EWMA). The primary aim of MEWMA is to quickly identify small variations that are present in a process more rapidly than the Hoteling's T^2 and MCSUM methods for the fact that the charting scheme takes advantage of the knowledge from previous observations in any given process. In other words, the MEWMA control chart is good at detecting small shift in the mean vector (Lowry *et al*., 1992). The chart uses the charting statistic:

$$T_i^2 = Z_i' \Sigma_{Z_i}^{-1} Z_i ; i = 1,2, \ldots \quad (1)$$

where

$$Z_i = \lambda x_i + (1-\lambda) Z_{i-1} \quad (2)$$

and the covariance matrix is given by

$$\Sigma_{Z_i} = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\Sigma \quad (3)$$

with the scalar charting constant λ, $0 < \lambda \le 1$ (which may be adjusted to change the weighting of the past observation). Z_i is the vector of observations at time i when $Z_0 = 0$. The MEWMA control chart often performs poorly when normal assumption is violated with common values of the charting constant greater than 0.05. The values acceptable for the charting constant are often very small, which means putting the majority of the weight on the past observations (instead of the most current), (Stoumbos and Sullivan, 2002; Lee and Khoo, 2006; Joner Jr. *et al*., 2008). A new maximum exponentially weighted moving average control chart for monitoring process mean and dispersion was developed by Rabyk and Schmid (2016). An extended nonparametric exponentially weighted moving average sign control chart

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was proposed by Lu (2015). New exponentially weighted moving average control charts for monitoring process dispersion was introduced by Haq *et al*. (2015). The EWMA method for detecting mean differences of a long-memory process was studied by Brown and Moltchanova (2015).

Generally, most parametric multivariate quality control methods usually adopted in determining if a system is performing as intended or if there are some unnatural causes of variation upon an overall statistics. It has the benefit of being able to handle different quality characteristics at the same time. However, the problems involve in the use of parametric multivariate method is the violation of assumption of multivariate normality that is required for many charts (Jarrett and Pan, 2007; Champ and Aparisi, 2007). To tackle this problem, the non-parametric kernel density estimation (KDE) Hotelling's T^2 control method was proposed by Chou *et al.* (2001). Hotelling's T^2 control limits obtained from this method is carried out without considering the assumption of normality. But the problem with the use of KDE is that it desires determination of numerous statistics or parameters to have a complete construction of the method. These consist of kernel functions, smoothing statistic, as well as mathematical integration to established kernel distribution so as to obtain the control limit.

To address these limitations (Phaladiganon *et al*., 2011) proposed the percentile bootstrap method as a means of obtaining Hoteling's T^2 control limits assuming that the distribution is not multivariate normal. However, this method bootstrapped from Hotelling's T^2 statistic obtained by collapsing the multivariate data into univariate, and this will results to control limits that is good in detecting of large shift only. Also, this method did not address the issue of out of control signal, hence the proposed methods. Like the normal Hotelling's T^2 method, existing methods are good in detecting large shift but insensitive to small and moderate shifts in process mean vector. To reduce the problem of violating multivariate distributional assumption as well as avoiding the problem of not detecting small to moderate shifts in the process mean vector (Chou *et al*., 2001; Phaladiganon *et al*., 2011; Chatterjee, and Qiu, 2009; Adewara and Adekeye, 2012), this study proposes the bootstrap multivariate exponentially weighted moving average (BMEWMA) for obtaining control limit. The proposed method is a distributional free assumption based on the non-parametric control chart.

MATERIALS AND METHODS

Proposed Bootstrap Multivariate Exponentially-Weighted Moving Average (BMEWMA) for obtaining Control Limit

To reduce the abnormal behaviours observed when the multivariate distributional assumption is violated as well as detecting small to moderate shift in any process; this study proposes the following procedure to obtain bootstrap multivariate exponentially weighted moving average control limits. Suppose there are ***d*** quality characteristics and each of the quality characteristic contains ***n*** set of observations (x_{ij}); $(i = 1,2,\dots,n; j = 1,2,\dots,d)$ as can be summarized in the matrix below.

$$\begin{matrix} x_1 & x_2 & \cdots & x_d \end{matrix}$$
$$\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1d} \\ x_{21} & x_{22} & \cdots & x_{2d} \\ & \cdots & & \\ x_{n1} & x_{n2} & \cdots & x_{nd} \end{pmatrix}_{dxn} \quad (4)$$

If the matrix notations of dxn dimensions can be transposed as expressions bellow:

$x_1 = (x_{11}, x_{21}, \dots, x_{n1})'$;
$x_2 = (x_{12}, x_{22}, \dots, x_{n2})'; \dots;$
$x_d = (x_{1d}, x_{2d}, \dots, x_{nd})'$

The proposed bootstrap **BMEWMA** procedure for obtaining control limits is as follows:

STEP 1. Combine the sample sizes of $x_1, x_2, \dots, x_d$ of the sets of observation such that:

$x = (x_{11}, x_{21}, \dots, x_{n1}; \dots; x_{1d}, x_{2d}, \dots, x_{nd})$

STEP 2. Draw a bootstrap sample of size $x^* = x_1^*, \dots, x_d^*$ with replacement in Step (1) as:

$x_{11}^*, x_{21}^*, \dots, x_{n1}^*; x_{12}^*, x_{22}^*, \dots, x_{n2}^*; \dots; x_{1d}^*, x_{2d}^*, \dots, x_{nd}^*$

STEP 3. Repeat Step (2) a large number of times and obtain bootstrap replications as:

$x^* = x_{11}^{*(i)}, x_{21}^{*(i)}, \dots, x_{n1}^{*(i)}; x_{12}^{*(i)}, x_{22}^{*(i)}, \dots, x_{n2}^{*(i)}; \dots; x_{1d}^{*(i)}, x_{2d}^{*(i)}, \dots, x_{nd}^{*(i)},$

where $(i^* = 1,2,\dots,B)$, and $B > 1000$.

STEP 4. Estimate the bootstrap replication mean vector $(\bar{x}^*)$, bootstrap replication variance and covariance matrix (S^*) of the bootstrap sample variables in Step (3)

STEP 5. Obtain **BMEWMA** (T_i^{2*}) statistics from the dataset in Step (4) as

$$T_i^{2*} = Z_i^{*'} \Sigma_{Z_i^*}^{-1} Z_i^*, \quad (5)$$

where

$$Z_i^* = \lambda x_i^* + (1-\lambda) Z_{i-1}^* \quad (6)$$

$$\Sigma_{Z_i^*} = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\Sigma^* \; ; 0 < \lambda \leq 1 \quad (7)$$

STEP 6. For $\mathbf{B = 3000}$, repeat the processes in Step (5) 3000 times by changing the values of Z_B & Σ_{Z_B} appropriately to obtain $T_1^{2*}, T_2^{2*}, \dots, T_B^{2*}$.

STEP 7. Set the upper control limit such that in each of the bootstrap statistic $(T_1^{2*}, T_2^{2*}, \dots, T_B^{2*})$ arranged from the lowest to highest figure, determine the position of $B(1-\alpha)^{th}$ value such that:

$$CL_{prop.bmewma.} = \frac{1}{B} \#\{(T_1^{2*}, T_2^{2*}, \dots, T_B^{2*}) \leq B(1-\alpha)\} \quad (8)$$

STEP 8. From the control limit established in Step (7), determine those quality characteristics that are under control process from those that are out of control process. That is, if any T^2 statistic is greater than $CL_{prop.bmewma.}$, declare that particular observation as out of control.

Proposed P-values Method in Identifying Out of Control Signals

The problem of identifying quality characteristic(s) that is(are) responsible for out of control signal(s) has been an issue in multivariate control charts (Runger *et al.*, 1996; Mason *et al.*, 1995; 1997; Das 2006; Guh 2007; Li *et al.*, 2013). A very useful approach in identifying out of control signal is to obtain the p-values of the Hotelling's T^2 statistic that reflect the contribution of each variable. Adopting Mason *et al.*, 1995, Step 1 and 2 were obtained while Step 3 and 4 where introduced to obtain their p-values.

STEP 1. For a d-dimensional vector of quality characteristics, the first row is expressed as:

$$T^2 = T^2_{j.i;\ \forall j=1,i=j-1},\ T^2_{j.i;\ \forall j=2,i=j-1,j-2},\ \cdots,\ T^2_{j.i;\ \forall j=d,i=j-1,j-2,j-3,\dots,j-d}$$

$$= T^2_1, T^2_{2.1}, T^2_{3.1,2}, T^2_{4.1,2,3}, \dots, T^2_{d.1,2,3,\dots,d-1}$$

STEP 2. Obtain f-distribution for each of T_j^2 and $T_{j.i}^2$ terms such that:

$$T_j^2 \sim \frac{c(n+1)(n-1)}{n(n-c)} f_{(c,n-c,\alpha)},\ c = 1$$; and

$$T_{j.i}^2 \sim \frac{c(n+1)(n-1)}{n(n-c)} f_{(c,n-c,\alpha)},\ c = 2, 3, \dots, j-1$$

are used to check if the j^{th} quality characteristic is conforming to the association with other quality characteristics or not.

STEP 3. Repeat Steps 1 and 2 for other rows based on the number of quality characteristics (d!) and obtain the distinct terms ($d*2^{d-1}$) for both the unconditional (T_j^2) and conditional ($T_{j.i}^2$) terms.

STEP 4. Obtain the bootstrap p-values for eachof T_j^2 and $T_{j.i}^2$ terms such that:

where $P_{value(Exist.Boot.)}$ denotes the p-value from the existing method and $P_{value(BMEWMA)}$ denotes the p-value from the proposed method.

STEP 5. Use the various P_{values} in Step 4 to assess whether there is a significant difference or not. If $(P_{values(Exist.Boot,\ BMEWMA)} > \alpha)$ value, it means that T_j^2 or $T_{j.i}^2$ is (are) not responsible for the out of control signal(s). But when $(P_{values(Exist.Boot,\ BMEWMA)} \leq \alpha)$ value, it means that T_j^2 or $T_{j.i}^2$ is (are) responsible for the out of control signal(s).

Application to Numerical Example

By way of illustration, the bootstrap procedure presented is implemented for a set of data with equal sample sizes. The set of data were obtained from the production processing of Owel Industries Nig. Ltd., a Family Delight Pure Soya Oil Production Company in Ekpoma, Edo State, Nigeria. Four quality characteristics (X_1, X_2, X_3 and X_4) representing phosphoric acid (milliliters), water (liters), caustic soda solution (kg) and industrial salt (kg) respectively at the neutralizer stage, under which forty five samples were recorded as shown in Columns (X_1, X_2, X_3 and X_4) of Table 1. The choice of data used in this study is the presence of sub-standard product of cooking oil displayed in the local market in Nigeria. Another motivation is the challenges faced by Quality Control Managers to discover the quality characteristic that is liable for the abnormal control behaviors or stop the entire production process. Stopping the process will result to waste of material resources and continuing with the process without identifying the variable will lead to sub-standard product. The urge to solve these problems gave rise to this research work.

Test of Correlation Coefficient

Nevertheless, to apply any multivariate control chart methodology, there is need to know whether there is association among the four variables. From the given data, the statistics of mean vector ($\bar{x}$), variance covariance matrix (s) and correlation matrix (r) are given as follows:

$$\bar{x} = \begin{bmatrix} 2451.2222 \\ 89.0889 \\ 26.6444 \\ 5.5489 \end{bmatrix},$$

$$s = \begin{bmatrix} 186363.813 & 172.389 & 370.672 & -16.141 \\ 172.389 & 101.674 & 1.260 & -2.027 \\ 370.672 & 1.260 & 7.962 & -0.112 \\ -16.141 & -2.027 & -0.112 & 0.269 \end{bmatrix} \text{ and}$$

$$r = \begin{bmatrix} 1.0000 & 0.040 & 0.304^{*} & -0.072 \\ 0.040 & 1.000 & 0.044 & -0.388^{**} \\ 0.304^{*} & 0.044 & 1.0000 & -0.076 \\ -0.072 & -0.388^{**} & -0.076 & 1.0000 \end{bmatrix}$$

* Significant at 0.05 ** Significant at 0.01

The association matrix denoted that there is relationship among the variables, hence the need for implementing multivariate control chart method. The value of Hotelling's T^2 statistic is computed using the Visual Basic

Table 1. Family Delight Pure Soya Oil Production Data and T^2 Statistic

Sample	X_1	X_2	X_3	X_4	T^2	Sample	X_1	X_2	X_3	X_4	T^2	Sample	X_1	X_2	X_3	X_4	T^2
1	3000	94	30	5.3	2.5324	**16**	1050	70	20	6.2	**15.6408**	**31**	2450	88	24	5.3	1.3496
2	2850	90	28	5.6	0.9443	**17**	3000	82	30	6	3.5346	**32**	2680	96	26	4.9	2.0374
3	2300	92	24	5.4	1.0588	**18**	2850	80	30	5.2	3.7166	**33**	2750	100	22	6	7.3517
4	2500	80	25	5.2	2.499	**19**	2000	95	31	5	5.9975	**34**	2900	87	29	6.3	3.9443
5	2750	45	27	7.5	**24.9818**	**20**	2050	86	26	5.8	1.0593	**35**	2850	89	30	5.1	2.4777
6	2400	82	26	5.8	0.581	**21**	2150	91	25	5.7	0.803	**36**	2000	96	26	5.3	1.7141
7	1550	80	20	5.1	**10.5469**	**22**	2060	83	28	5.4	2.1454	**37**	3000	99	27	6.1	5.2445
8	2950	100	30	4.2	**8.2502**	**23**	2700	90	24	5.6	1.723	**38**	2150	100	28	6	5.9041
9	2850	93	29	6.1	3.271	**24**	2800	94	25	5.3	1.7413	**39**	2300	101	22	5.8	5.1479
10	2300	85	25	5.9	0.7641	**25**	2950	85	27	5.4	1.7658	**40**	2400	102	25	5.7	2.717
11	2250	95	24	5.5	1.3214	**26**	2250	86	29	5.4	1.549	**41**	2600	80	28	5.2	2.2793
12	2900	80	26	5.2	3.4108	**27**	2005	97	32	5.9	**8.2599**	**42**	2015	94	29	5.9	3.5712
13	2550	87	27	5.7	0.1649	**28**	2010	100	24	5.6	3.0063	**43**	2225	90	30	6	3.3235
14	2100	98	28	5.4	2.0423	**29**	3010	98	23	5	6.2249	**44**	2450	98	27	5.4	0.7971
15	2000	86	29	5	4.3045	**30**	2500	84	28	4.8	3.5793	**45**	2900	81	26	5.5	2.2492

Table 2. Phaladigalon's Bootstrap Method Replicated from T^2 Statistic.

S/N	**Bootstrap T^2 Replicated**											Percentile value
	1	**2**	**3**	**4**	**5**	**. . .**	**41**	**42**	**43**	**44**	**45**	**100(1-α)**
1	3.41	2.04	5.99	5.15	8.25	. . .	8.25	2.25	2.28	3.41	6.22	10.09
2	4.30	2.245	3.94	3.32	3.27	. . .	8.25	0.58	2.28	1.71	1.32	8.258
3	5.90	6.22	1.72	2.15	1.55	. . .	5.245	1.74	3.32	3.01	0.16	8.071
4	1.55	7.35	0.80	0.80	6.22	. . .	2.532	0.94	6.22	1.72	2.5	14.62
.	.	.	.	.	.		.	.	.	.	.	.
.	.	.	.	.	.	. . . .	.	.	.	.	.	.
.	.	.	.	.	.		.	.	.	.	.	.
2997	1.741	1.35	2.25	3.94	5.9	. . .	3.944	2.04	3.57	0.76	0.58	8.25
2998	5.245	3.72	2.15	8.26	5.15	. . .	5.148	3.57	5.9	3.58	6.22	15.64
2999	2.249	5.9	4.3	1.06	3.94	. . .	3.271	1.77	3.32	3.94	3.27	23.11
3000	4.305	2.28	3.72	5.24	3.41	. . .	1.321	0.8	6.22	1.06	0.76	6.179

Code (F-Distribution Form Code in the Appendix) for each sample and summarized in Table 1.

Using the existing method, bootstrap samples are replicated 3000 times from the Hotelling's T^2 statistic in Table 1, and 100(1-α) percentile value computed for each sample is shown in Table 2.

However, the proposed bootstrap multivariate exponential weighted moving average (BMEWMA) procedures was implemented in Visual Basic Code (MEWMA Bootstrap Form Code in the Appendix) to obtain the bootstrap sample replicated 3000 times from the original data set and BMEWMA T^2 statistic is computed for each as shown in Table 3.

Table 4 shows the results of control limits obtained at α = 0.05 level of significant from f-distribution, existing bootstrap method and Step (9) from the proposed **BMEWMA** method, and the control chart is shown in Figure 1.

Identification of Out of Control Signal by P-value Method for Sample 16

Figure 1 shows that samples 5,7,8,16 and 27 are out of control, but we do not know which or set of quality characteristic(s) that is(are) responsible for the signal(s), hence the need to identify those quality characteristics by using the proposed p-value method. Focusing on Sample 16 by repeating Steps (1-4), Table 5 shows all the unconditional and conditional T^2 values and compared with their various p-values.

Table 3. Bootstrap Sample Replicated and BMEWMA T^2 Statistic.

Sample	Y1	Y2	Y3	Y4	Z1	Z2	Z3	Z4	T^2	T^2 Sorted
1	69.8891	-3.6446	0.9116	-0.0623	6.99	-33.26	0.75	-0.25	5.136	0.046
2	2.8891	-0.2446	0.2004	-0.1623	63.19	-3.3	0.84	-0.07	10.761	0.047
3	57.6669	2.2888	0.6671	-0.1268	8.37	0.01	0.25	-0.16	4.644	0.113
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
2850	-27.4442	-0.7334	-0.0884	0.0021	48.86	-0.47	0.57	-0.11	4.512	**7.406**
2851	-49.3331	-1.3557	-0.7551	-0.0379	52.77	-3.34	-0.26	0.05	6.523	7.415
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
2998	24.1113	-1.2668	-0.3329	-0.0223	-31.69	-2.35	-0.15	0.03	2.934	16.732
2999	-34.222	-0.2223	0.0671	0.051	70.98	-2.06	0.23	0	3.319	19.757
3000	-36.5553	0.7332	-0.2662	-0.0823	26.84	1.91	0.25	-0.04	2.401	24.534

Table 4. Control Limits for the Different Methods at α Level of 0.05.

Alpha level (α)	F-Distribution	Existing Bootstrap Method	Proposed BMEWMA Method
0.05	11.4089	11.3314	7.406

RESULTS AND DISCUSSION

The value of control limit for f- distribution is given by $CL_F = 11.4089$, existing bootstrap method gives a $CL_{Exist.Boot} = 11.3314$ and that of proposed BMEWMA method gives $CL_{BMEWMA} = 7.406$ assuming α = 0.05. Comparing these control limits in Table 4 with column T^2 in Table 1, out of control signals was detected when Samples 5, 7, 8, 16 and 27 were considered, and their positions are shown in Figure 1. Results in Table 4 shows that control limits obtained from both f-distribution and existing bootstrap methods were able to detect out of control signals for Samples 5 and 16, but consider Samples 7, 8 and 27 to be under control. This has demonstrated the ability of the two Hotelling's T^2 methods to detect large shift in the process mean vector. However, the proposed BMEWMA method was able to detect out of control signals in additional three Samples (7, 8 and 27), this has demonstrated the ability of the proposed method to detect small to moderate shift in any process.

From Table 5a, T_1^2 *and* T_3^2 of the four unconditional T^2 terms associated with Sample 16 are significant, which means X_1 (phosphoric acid) and X_3 (caustic soda) are responsible for the out of control signals individually. However, results from existing p-values did not support this finding. To reduce the problem of out of control signal facing variables X_1 and X_3, remove T_1^2 *and* T_3^2 separately from $T^2 = 15.6408$ of Sample 16 and compare with the control limits whether they are significant or not. i.e.

$$T^2 - T_2^2 = 15.6408 - 10.5354 = 5.1054 < 11.4689, 11.3336, 7.415$$

Hence, we conclude that variable X_1 is not significant. However, result obtain when T_3^2 is removed from T^2 shows that X_3 (caustic soda) is significant when compared with Control Limits from the proposed methods in Table 4, hence the next step, i.e.

$$T^2 - T_2^2 = 15.6408 - 5.5435 = 10.1063 < 11.4689, 11.3336 > 7.415$$

In Table 5b, the first conditional T^2 terms associated with Sample 16 shows that $T_{1.2}^2, T_{1.3}^2, T_{1.4}^2$ of the twelve conditional T^2 terms have significant values, which means the relationship between X_1 (phosphoric acid) and X_2 (water); X_1 (phosphoric acid) and X_3 (caustic soda); X_1 (phosphoric acid) and X_4 (industrial salt) respectively are responsible for out of control signals. To reduced the problem of out of control signal facing these 1st conditional variables, remove $T_{1.2}^2$, $T_{1.3}^2$, $T_{1.4}^2$ separately from $T^2 = 15.6498$ of Sample 16 and compare

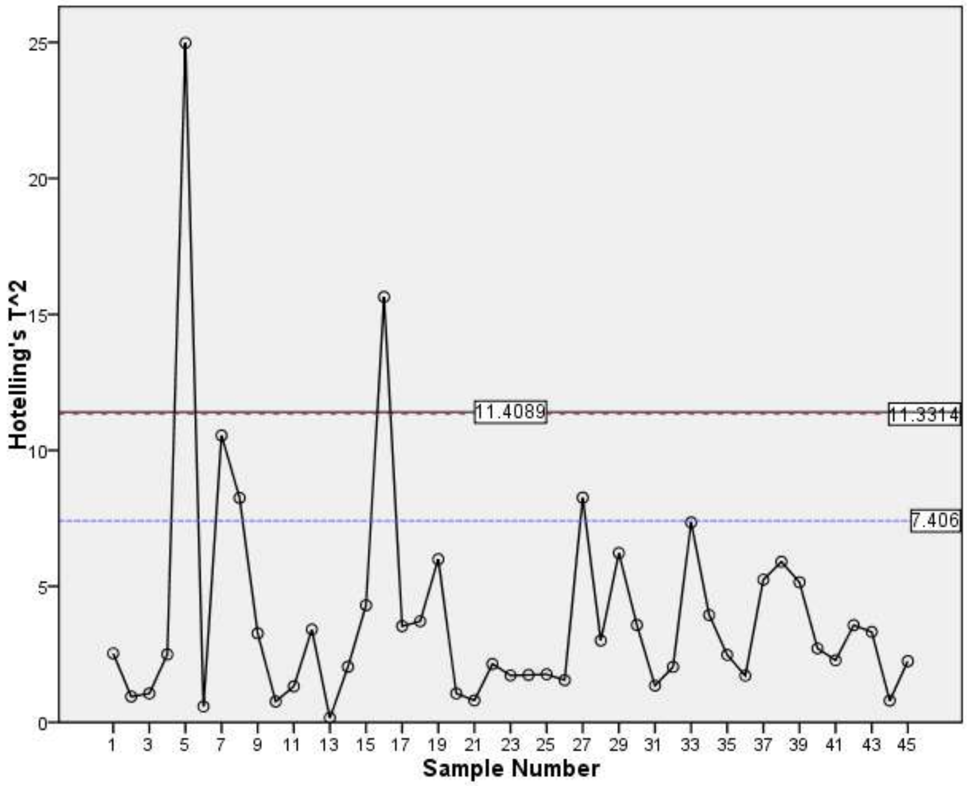

Fig. 1. Multivariate Hotelling's T^2 Control Chart for the given Data

Table 5. Bootstrap P-values for Unconditional and Conditional T^2 Terms for Sample 16

Table 5a. Unconditional T^2 Terms with p-values (Number of $T^2_{Sortd} \geq T^2_j$ in Parenthesis)

T_j^2 Component	Computed T_j^2 Value	F- Critical values	Existing Bootstrap P-Value	Proposed BMEWMA P-value	T_j^2 Reduced (Sig./Not)
T_1^2	10.5354*	4.1519	0.421 **(1263)**	0.0127** **(38)**	5.1054 (N/S)
T_2^2	3.5839	„	1.000 **(3000)**	0.3243 **(973)**	N/S
T_3^2	5.5435*	„	0.9973 **(2992)**	0.1263 **(379)**	10.1063 (Sig)
T_4^2	1.5760	„	1.000 **(3000)**	0.7407 **(2222)**	N/S

with the control limits whether they are significant or not. $T^2_{1.2}$, $T^2_{1.4}$ are not significant as shown in the last columns of Table 5b, while $T^2_{1.3}$ is significant, hence we move to the next step, i.e.

$$T^2 - T^2_{1.2} = 15.6408 - 10.0702 = 5.5706 < 11.4689, 11.3336, 7.415$$

$$T^2 - T^2_{1.3} = 15.6408 - 7.0501 = 8.5907 < 11.4689, 11.3336 > 7.415$$

$$T^2 - T^2_{1.4} = 15.6408 - 9.9047 = 5.7361 < 11.4689, 11.3336, 7.415$$

In Table 5c, the second conditional T^2 terms associated with Sample 16 shows that $T^2_{1.24}$ of the twelve conditional T^2 terms has significant value, which means the relationship between X_1 (phosphoric acid) and X_2 (water) and X_4 (industrial salt) are responsible for the out of control signals. To reduce the problem facing 2nd conditional variables, remove $T^2_{1.24}$, from $T^2 = 15.6498$ of Sample 16 and compare with the control limits whether it is significant or not. $T^2_{1.24}$ is not significant as shown in the last columns of Table 5c, hence we stop the process, i.e.

$$T^2 - T^2_{1.24} = 15.6408 - 9.8877 = 5.7531 < 11.4689, 11.3336, 7.415$$

Since there is no more difference in Table 5c, Table 5d has no significant difference.

Table 5b. 1st Conditional T^2 Terms with p-values (Number of $T^2_{Sortd} \geq T^2_{j.i}$ in Parenthesis).

$T^2_{j.i}$ Component	Computed $T^2_{j.i}$ Value	F- Critical values	Existing Bootstrap P-Value	Proposed BMEWMA P-value	$T^2_{j.i}$ Reduced (Sig./Not)
$T^2_{1.2}$	10.0702*	6.7247	0.5983 **(1795)**	0.0147** **(44)**	5.5706 (N/S)
$T^2_{1.3}$	7.0501*	,,	0.95 **(2850)**	0.0607 **(182)**	8.5907 (Sig)
$T^2_{1.4}$	9.9047*	,,	0.5793 **(1738)**	0.015** **(45)**	5.7361 (N/S)
$T^2_{2.1}$	3.1186	,,	1.000 **(3000)**	0.3977 **(1193)**	N/S
$T^2_{2.3}$	3.2062	,,	1.000 **(3000)**	0.3833 **(1150)**	N/S
$T^2_{2.4}$	1.7200	,,	1.000 **(3000)**	0.7067 **(2120)**	N/S
$T^2_{3.1}$	2.0595	,,	1.0000 **(3000)**	0.6183 **(1855)**	N/S
$T^2_{3.2}$	5.1672	,,	0.9983 **(2995)**	0.1503 **(451)**	N/S
$T^2_{3.4}$	5.0719	,,	0.9987 **(2996)**	0.1603 **(481)**	N/S
$T^2_{4.1}$	1.0378	,,	1.000 **(3000)**	0.8613 **(2584)**	N/S
$T^2_{4.2}$	0.2366	,,	1.000 **(3000)**	0.9943 **(2983)**	N/S
$T^2_{4.3}$	1.1492	,,	1.000 **(3000)**	0.8397 **(2519)**	N/S

*Out of Control Signals **Significant at 0.05 N/S (Not Significant) (Sig) Significant ($T^2_{j.i} > CL$)

CONCLUSION

This study specifically considered the BMEWMA method as a means of determining control limits from multivariate control charts. The intension is to reduce the rate of not discovering false alarm in any given process. Procedures that can carry out a systematic generation of bootstrap replications for two or more quality characteristics have been proposed in this work; it is straight forward but computer intensive. Using numerical example, control limits obtained from the proposed method performed very well when compared with the existing methods as shown in Table 4 and Figure 1, i.e. $CL_F = 11.4089$, $CL_{ExistBoot} = 11.3314$ and $CL_{BMEWMA} = 7.406$; thereby ensuring reduction in the rate of not discovering false alarm. However, to identify the root cause of change when multivariate control charts signals, this work also considered the p-value as a means of identifying the variable(s) that is(are) responsible for the out of control signal(s). From Table 5a, computed T^2_3 value is greater than F-critical value (i.e. 5.5435 > 4.1519); existing bootstrap p-value is greater than α value (i.e. 0.9973 > 0.05); and proposed bootstrap p-value is greater than α value (i.e. 0.1263 > 0.05), then the next stage in Table 5b. From Table 5b, computed $T^2_{1.3}$ value is greater than F-critical value (i.e. 7.0501 > 6.7247); existing bootstrap p-value is greater than α value (i.e. 0.95 > 0.05); and proposed bootstrap p-value is greater than α value (i.e. 0.0607 > 0.05), hence the next stage in Table 5c. In Table 5c, computed $T^2_{1.24}$ value is greater than F-critical value (i.e. 9.8877 > 9.0824); existing bootstrap p-value is greater than α value (i.e. 0.5793 > 0.05); and proposed bootstrap p-value is less than α value (i.e. 0.0153 > 0.05), hence no more significant value as experienced in Table 5d. Results from Tables 5a, b, c, d showed that the major problem lies on variable X_1 (phosphoric acid) and X_3 (caustic soda) for Sample 16. The usual control chart practice method is to stop the entire process as a result of out of control signal at variable X_1 and X_3, this will result to waste of material resources or low quality/sub standard products if the process continuous. With the proposed multivariate methods, one variable is being conditioned on the other(s) as shown in Tables 5a, b, c, d. The implication of these findings is the advantages of multivariate control charts; by combining variable X_1 or X_3 with any other variables until there is no out of control signals as observed in Table 5d. This finding will enhance production process and avoid waste of material resources and improve the quality of product.

Table 5c. 2nd Conditional T^2 Terms with p-values (Number of $T^2_{Sortd} \geq T^2_{j.i}$ in Parenthesis).

$T^2_{j.i}$ Component	Computed $T^2_{j.i}$ Value	F- Critical values	Existing Bootstrap P-Value	BMEWMA P-value	$T^2_{j.i}$ Reduced (Sig./Not)
$T^2_{1.23}$	6.7966	9.0824	0.9517 **(2855)**	0.0713 **(214)**	N/S
$T^2_{1.24}$	9.8877*	,,	0.5793 **(1738)**	0.0153** **(46)**	5.7531 (N/S)
$T^2_{1.34}$	6.7769	,,	0.9517 **(2855)**	0.0713 **(214)**	N/S
$T^2_{2.13}$	2.9527	,,	1.000 **(3000)**	0.4267 **(1280)**	N/S
$T^2_{2.14}$	2.2076	,,	1.000 **(3000)**	0.5827 **(1748)**	N/S
$T^2_{2.34}$	2.2200	,,	1.000 **(3000)**	0.5803 **(1741)**	N/S
$T^2_{3.12}$	1.8936	,,	1.0000 **(3000)**	0.6593 **(1978)**	N/S
$T^2_{3.14}$	1.9004	,,	1.0000 **(3000)**	0.6587 **(1976)**	N/S
$T^2_{3.24}$	5.0236	,,	0.999 **(2997)**	0.164 **(492)**	N/S
$T^2_{4.12}$	0.1377	,,	1.000 **(3000)**	0.9983 **(2995)**	N/S
$T^2_{4.13}$	0.8895	,,	1.000 **(3000)**	0.8927 **(2678)**	N/S
$T^2_{4.23}$	0.1766	,,	1.000 **(3000)**	0.9967 **(2990)**	N/S

*Out of Control Signals **Significant at 0.05 N/S (Not Significant) (Sig) Significant ($T^2_{j.i} > CL$)

Table 5d. 3rd Conditional T^2 Terms with p-values (Number of $T^2_{Sortd} \geq T^2_{j.i}$ in parenthesis).

$T^2_{j.i}$ Component	Computed $T^2_{j.i}$ Value	F-Critical values	Existing Bootstrap P-Value	BMEWMA P-value	$T^2_{j.i}$ Reduced (Sig./Not)
$T^2_{1.234}$	0.8247	11.4088	1.0000 (3000)	0.906 (2718)	N/S
$T^2_{2.134}$	5.0433	,,	0.5650 (1695)	0.141 (423)	N/S
$T^2_{3.124}$	0.0293	,,	1.0000 (3000)	1.000 (3000)	N/S
$T^2_{4.123}$	5.3164	,,	0.9977 (2993)	0.141 (423)	N/S

*Out of Control Signals **Significant at 0.05 N/S (Not Significant) (Sig) Significant ($T^2_{j.i} > CL$)

REFERENCES

Adewara, JA. and Adekeye, KS. 2012. Bootstrap T^2 technique for minimax multivariate Control chart. Open Journal of Statistics. 2:469-473.

Aparisi, F., Champ, CW. and Garcia – Diaz, JC. 2004. A performance analysis of Hotelling's T^2 control chart with supplementary runs rules, Quality Engineering. 16(3): 359-368.

Brown, J. and Moltchanova, E. 2015. New exponentially weighted moving average control charts for monitoring process mean and process dispersion. Quality and Reliability Engineering International. 31(5):877-901.

Champ, CW. and Aparisi, F. 2007. Double sampling Hotelling's T^2 chart. Quality and Reliability Engineering International. 24:153- 166.

Champ, CW. and Jones-Farmer, LA. 2007. Properties of multivariate control charts with estimated parameters. Sequential Analysis. 26:153-169.

Chatterjee, S. and Qiu, P. 2009. Distribution-free cumulative sum control charts using bootstrap based control limits. The Annals of Applied Statistics. 3(1):349-369.

Chou, YM., Manson, RL. and Young, JC. 2001. The control chart for individual observations from a multivariate non-normal distribution. Communications in Statistics Simulation and Computation. 30(8-9):1937-1949.

Das, N. 2006. Multivariate diagnostic tests, a review of different methods. Economic Quality Control. 21(1):143-153.

Haq, A., Brown, J. and Moltchanova, E. 2015. A new maximum exponentially weighted moving average control chart for monitoring process mean and dispersion. Quality and Reliability Engineering International. 31(8):1587-1610.

Jarrett, JE. and Pan, X. 2007. The quality of control chart for monitoring multivariate autocorrelation process. Computational Statistics and Data Analysis. 51:3862-3870.

Joner Jr., MD., Woodall, WH., Reynolds Jr., MR. and Fricker Jr. RD. 2008. A one sided MEWMA chart for health surveillance. Quality and Reliability Engineering International. 24: 503-518.

Lee, MH. and Khoo, M. 2006. Optimal statistical design of a multivariate EWMA chart based on ARL and MRL. Communication in Statistics-Simulation and Computation. 35: 831-847.

Li, T., Hu, S., Wei, Z. and Liao, Z. 2013. A framework for diagnosing the out of control signals in multivariate process using optimized support vector machines. Mathematical Problems in Engineering in http://dx.doi.org/10.1155/2013/494626.

Lowry, CA., Woodall, WH., Champ, CW. and Rigdon, SE. 1992. A Multivariate exponential weighted moving average control chart. Technometrics. 34:46-53.

Lu, SL. 2015. An extended nonparametric exponentially weighted moving average sign control chart. Quality and Reliability Engineering International. 31(1):3-13.

Mason, RL., Tracy, ND. and Young, JC. 1995. Decomposition of multivariate control chart interpretation. Journal of Quality Technology. 27:99-108.

Mason, RL., Tracy, ND. and Young, JC. 1997. A practical approach for interpreting multivariate control chart signals. Journal of Quality Technology. 29:396-406.

Montgomery, DC. 2009. Introduction to statistical quality control (6th edi.). John Wiley and Sons, New Jersey, USA. 499-513.

Mahmoud, MA. and Maravelakis, PE. 2010. The performance of the MEWMA control chart when parameters are estimated. Communication in Statistics-Simulation and Computation. 39(9): 1803-1817. doi:10.1080/03610918.2010.518269

Phaladiganon, P., Kim, SB., Chen, VCP., Baek, JG. and Park, SK. 2011. Bootstrap based T^2 multivariate control charts. Communication in Statistics-Simulation and Computation. 40(5): 645-662.

Rabyk, L. and Schmid, W. 2016. EWMA control charts for detecting changes in the mean of a long-memory process. Metrika 79: 267-301. doi:10.1007/s00184-015-0555-7.

Runger, GC., Alt, FB. and Montgomery, DC. 1996. Contributors to a multivariate statistical process control signal. Communication in Statistics: Theory and Methods. 25(10):2203-2213.

Smiley, WC. and Keoagile, T. 2005. Multivariate Max-CUSUM Chart. Quality Technology & Quantitative Management. 2(2): 221-235.

Stoumbos, ZG. and Sullivan, JH. 2002. Robustness to non-normality of the multivariate EWMA control chart. Journal of Quality Technology. 34:260-276.

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APPENDIX. MULTIVARIATE BOOTSTRAP CONTROL SYSTEM
Splash Screen Form
================

SplashScreen Form code
=====================

```
Private Sub tmrDisplay_Timer()
Unload Me
mdiMain.Show
End Sub
```

Main Form
==========

Main Form Code
=============

```
Private Sub mnuAboutSoftware_Click()
frmAbout.Show
End Sub

Private Sub mnuBootstrap_Click()
frmBootstrap.Show
End Sub

Private Sub mnuBootstrapClearAll_Click()
On Error Resume Next
If formBoostrap = True Then DataEnv.ClearBoostrap
DataEnv.rsBoostrap.Requery
frmBootstrap.DataGrid.Refresh
End Sub

Private Sub mnuBootstrapRealMEWMA_Click()
frmMEWMABootstrap.Show
End Sub

Private Sub mnuCloseWindow_Click()
If ActiveForm Then Unload ActiveForm
End Sub
Private Sub mnuFDistributionClearAll_Click()
On Error Resume Next
If formFDistr = True Then DataEnv.ClearFDistribution
DataEnv.rsF_T.Requery
frmF_Distr.DataGrid.Refresh
End Sub

Private Sub mnuFDistributionEmptyRows_Click()
On Error Resume Next
If formFDistr = True Then DataEnv.ClearEmptyRows
DataEnv.rsF_T.Requery
frmF_Distr.DataGrid.Refresh
End Sub

Private Sub mnuFTDistribution_Click()
frmF_Distr.Show
End Sub

Private Sub mnuMEWMABootstrapClearAll_Click()
On Error Resume Next
If formMEWMABootstrap = True Then
DataEnv.ClearMEWMABootstrap
If formModifiedMEWMABootstrap = True Then
DataEnv.ClearModifiedMEWMABootstrap
DataEnv.rsMEWMABootstrap.Requery
End Sub

Private Sub mnuMEWMAClearAll_Click()
On Error Resume Next
If formRealMEWMA = True Then
DataEnv.ClearRealMEWMA
DataEnv.rsRealMEWMA.Requery
End Sub

Private Sub mnuRealMEWMA_Click()
frmRealMEWMA.Show
End Sub

Private Sub mnuSaveChanges_Click()
On Error Resume Next
With DataEnv
   .rsF_T.UpdateBatch adAffectAll
   .rsBoostrap.UpdateBatch adAffectAll
   .rsRealMEWMA.UpdateBatch adAffectAll
    .rsMEWMABootstrap.UpdateBatch adAffectAll
End With
End Sub
```

F-Distribution Form
================

F-Distribution Form Code

```
====================
Dim Max_n As Integer, response As Integer
Dim i, ii, iii, nCount As Integer
Dim mSum As Single, vSum As Single, Variance As
Single, Mean As Single
Dim covSum As Single, Covariance As Single
Dim MatrixLine(10, 10) As Single
Dim MatrixInverse(10, 10) As Variant
Option Explicit

Exit Sub
errorHandler:
MsgBox Err.Description, vbCritical
End Sub

Sub ClearTextboxes()
For n = 0 To 99
    Text1(n).Text = ""
Next n
End Sub

Private Sub DataGrid_Click()
txtTotalRecord.Text = DataGrid.ApproxCount
txtCurrentRecord.Text = DataGrid.Row + 1
End Sub

Private Sub Form_Resize()
On Error Resume Next
'resize datagrid
If Me.Width - DataGrid.Width > 0 Then
  DataGrid.Width = Me.Width - 9000
  DataGrid.Height = Me.Height - 1900
End If
End Sub


Private Sub Form_Unload(Cancel As Integer)
formFDistr = False
mdiMain.mnuFTDistribution.Checked = False
End Sub
```

MEWMA Bootstrap Form

====================

MEWMA Bootstrap Form Code

```
========================
Dim nCount, Max_n, Max_b, iCount
Dim genNum As Integer

Dim mSum As Single, vSum As Single, Variance As
Single, Mean As Single
Dim covSum As Single, Covariance As Single, preZi As
Single
Dim MatrixLine(10, 10) As Single
Dim MatrixInverse(10, 10) As Variant
Option Explicit


Private Sub cmdSOLVE_Click()
On Error Resume Next
  Call ComputeMean
  Call ComputeVarCov
  Call Build_Matrix
  Call Calculate_Inverse
  Call Calculate_Transpose
  Call Type_Result
  DataEnv.rsMEWMABootstrap.UpdateBatch
adAffectAll
End Sub

Private Sub DataGrid_Click()
txtTotalRecord.Text = DataGrid.ApproxCount
txtCurrentRecord.Text = DataGrid.Row + 1
End Sub
```